Synopsis for Ph.D. Programme

Title of Research:
A study on generalized fuzzy normed linear space and developing fixed point theory in such spaces

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Introduction

In 1965, Lotfi A. Zadeh first introduced the idea of fuzzy sets by which it has been possible to device mathematical models for those so-called vague objects such as imprecision, vagueness, insufficient information etc. The demand of fuzzy set has enormously been increased day-by-day for it's applicability in a variety of fields such as artificial intelligence, computer science, robotics, Pattern Recognition, Decision Making etc. So, fuzzification of mathematics is going on in different directions such as fuzzy set theory, fuzzy number theory, fuzzy topology, fuzzy functional analysis etc for it's wide applicability in real-life problems. It is well known that metric and norm structure play pivotal role in 'functional analysis'. So, in order to develop 'fuzzy functional analysis' one has to take care about the suitable fuzzifications of these structures.

1 Brief history of previous work:

Historically the problem of fuzzification of the metric structure came first. Different authors introduced ideas of fuzzy metric space, statistical fuzzy metric space, fuzzy pseudo metric space etc. In 1975, Kramosil and Michalek [1] introduced the concept of fuzzy metric space and studied some properties of such spaces. In 1984, Kaleva and Seikkala [2] introduced the definition of fuzzy metric space in a different approach and established Banach fixed point theorem in fuzzy setting. After that, George and Veeramani [3] proposed a stronger from of definition of fuzzy metric. In 1984, A. K. Katsaras [4] first introduced the idea of fuzzy norm on a linear space. In 1992, Felbin [5] introduced an idea of a fuzzy norm on a linear space whose associated metric is Kaleva [2] type. In 1994, Cheng and Mordeson [6] introduced a fuzzy norm on a linear space whose associated metric is Kramosil and Michalek [1] type. Following Cheng and Morderson [6], in 2003, Bag and Samanta [7] introduced a definition of fuzzy norm by taking t-norm as 'minimum' whose associated metric is similar to Kramosil and Michalek [1] type. They also established the decomposition theorem for this type of fuzzy norm into a family of crisp norms in [7]. In 2012, Bag and Samanta redefined fuzzy norm [8] by taking general t-norm and they developed some theory on this space such as l-fuzzy convergence, l-fuzzy closeness, l-fuzzy compactness, l-fuzzy completeness [9], operator's fuzzy norm [10] etc. In 2006, Z. Mustafa and B. Sims [11] introduced the concept of generalized metric space (G-metric space). In 2014, Karman Alam Khan introduced the concept of generalized norm [12] (G-norm).

2 Proposed Plan of work to be done:

From the research articles surveyed above, it appears that there is a lot of scope of future works in the field of fuzzy norm linear spaces under general t-norm context. More specifically I would like to study in the following directions:

- To study important features such as four fundamental theorems viz., the open mapping theorem, the closed graph theorem, the Hahn-Banach theorem, the uniform boundedness principle theorem on this space.

- To study fuzzy compact linear operator in generalized fuzzy norm linear space and to study some properties on it.
- To study some geometric structure such as uniform convexity, strictly convexity, normal structure, fixed point theory on this space.
- To establish some fixed point theorems for non-expansive mapping in fuzzy normed linear spaces.
- To fuzzify G-norm and to study some properties of fuzzified G-norm linear spaces.

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